Examples of anti-mating

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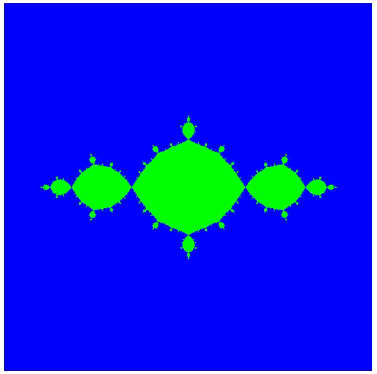
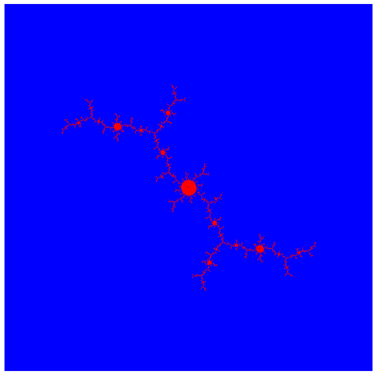
Recall mating (Douady, Hubbard, Rees, Shishikura, Tan):

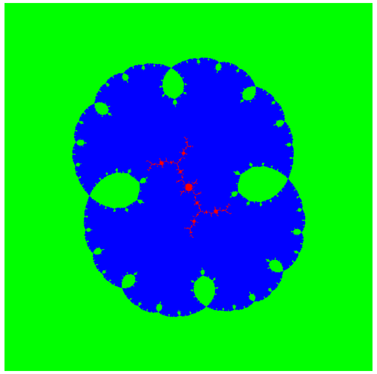
$Q(z) = z^2 + q$



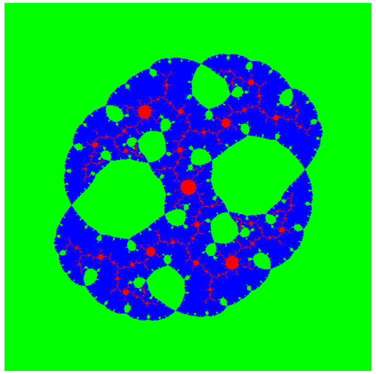
$P(z) = z^2 + p$



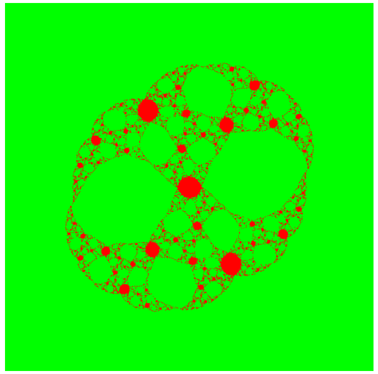
formal mating



Thurston pullback

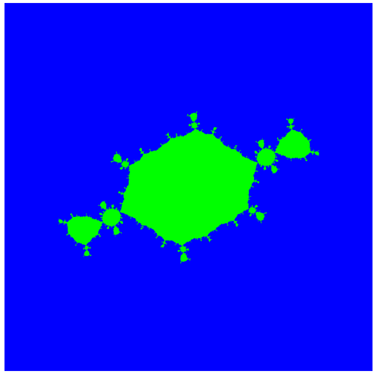


geometric mating



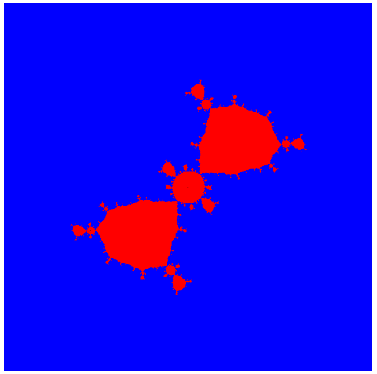
Anti-mating (Douady, Meyer, Timorin):

$P \circ Q(z)$

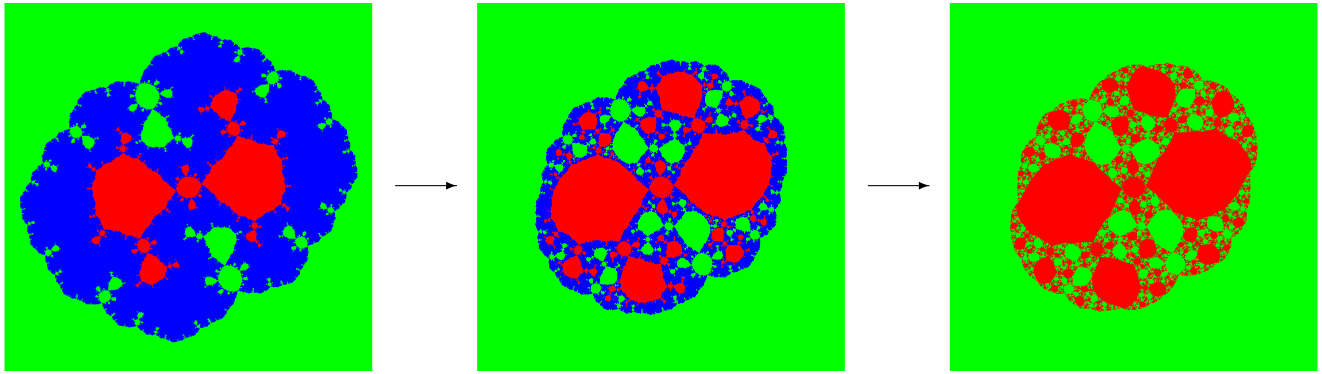


$P(z) = z^2 + p \uparrow \qquad Q(z) = z^2 + q \downarrow$

$Q \circ P(z)$



formal anti-mating $\longrightarrow$ Thurston pullback $\longrightarrow$ geometric anti-mating

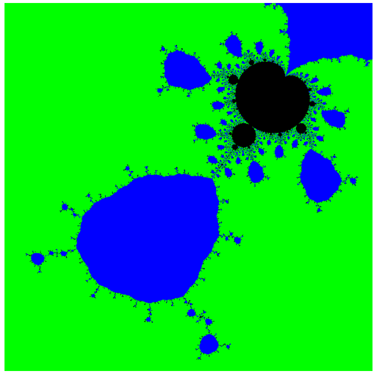


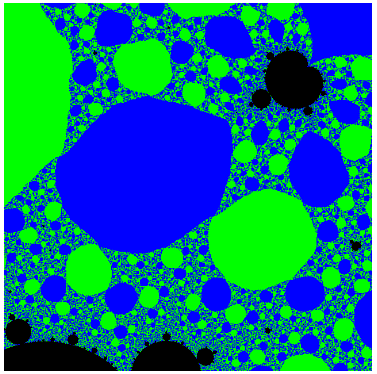
Explanation:

In the toy model, the quadratic polynomials $P(z) = z^2 + p$ and $Q(z) = z^2 + q$ are acting between two planes. $Q \circ P(z)$ and $P \circ Q(z)$ are symmetric quartic polynomials; they are semi-conjugate in both directions: $P \circ (Q \circ P) = (P \circ Q) \circ P$ and $Q \circ (P \circ Q) = (Q \circ P) \circ Q$. The two quartic filled Julia sets are mapped to each other by $P(z)$ and $Q(z)$. The toy model is pcf, iff $(P \circ Q)^n(0)$ and $(Q \circ P)^n(0)$ are finite, not $P^n(0)$ and $Q^n(0)$. The formal anti-mating is obtained by identifying the two planes with half-spheres; it may be Thurston-equivalent to a quadratic rational map $f(z)$, with suitable modifications in the case of removable obstructions. The topological anti-mating is defined by gluing $\mathcal{K}_{Q \circ P}$ and $\mathcal{K}_{P \circ Q}$ along their boundaries. Equipotential gluing is possible as well.

Example 1:

$p = -q^2$ gives $Q \circ P(z) = (z^2 - q^2)^2 + q$ and $P \circ Q(z) = z^4 + 2qz^2$. If the geometric anti-mating $f(z)$ of $P(z) = z^2 - q^2$ and $Q(z) = z^2 + q$ exists, it is in V_2: $f(z) = f_a(z) = \frac{z^2+a}{z^2-1}$ has a 2-perioidc critical point $\infty \Rightarrow 1 \to \infty$.
The images show corresponding regions of the connectedness locus in the q-plane and the a-plane V_2. Note: the geometric anti-mating $f_a(z)$ may be of **capture** type.





Example 2:

$p = 0$ gives $Q \circ P(z) = z^4 + q$ and $P \circ Q(z) = (z^2 + q)^2$. The connectedness locus in the q-plane is the Multibrot set of degree 4. If the geometric anti-mating $f(z)$ of $P(z) = z^2$ and $Q(z) = z^2 + q$ exists, it is of the form $f(z) = f_a(z) = 1 + \frac{a}{z^2}$. These quadratic rational maps are **bitransitive**: $\ldots \to 0 \Rightarrow \infty \Rightarrow 1 \to \ldots$

Basic results:

- If the formal anti-mating has only removable obstructions, the resulting quadratic rational map is conjugate to the topological anti-mating.
- Hyperbolic anti-matings are characterized by an anti-equator.
- For the symmetry locus $f_a(z) = \frac{z^2+a}{1+az^2}$ and $c \in \mathcal{M}$, $f_a(z)$ is the anti-mating of $z^2 + c$ with itself, iff $f_{1/a}(z)$ is the mating of $z^2 + c$ with itself. In particular, the anti-mating exists in the pcf case, iff c is not in the 1/2-limb.

Related projects:

- Determine loci of mating or anti-mating numerically.
- Convergence properties of slow mating and anti-mating.
- Canonical stratum in the removable case: show convergence to a suitable component of the noded moduli space.